Sesión Especial 6

Computación Simbólica: nuevos retos en Álgebra y Geometría y sus aplicaciones.

Organizadores

- Sonia Pérez Díaz (Universidad de Alcalá)
- Juana Sendra Pons (Universidad Politécnica de Madrid)
- Mª Pilar Vélez Melón (Universidad Antonio de Nebrija)

Descripción

El objetivo de esta sesión es ser un punto de encuentro para investigadores interesados en el estudio algorítmico de problemas de Álgebra y Geometría, incluyendo, además, nuevos ámbitos de aplicación surgidos de la ingeniería o la informática. La computación simbólica es la línea argumental de esta sesión, que pone el foco en cómo ésta puede integrarse en el tratamiento de problemas algorítmicos en Algebra y Geometría, y cómo estas aproximaciones computacionales tienen una aplicación directa en contextos tecnológicos fuera de las Matemáticas como pueden ser la informática (desarrollando herramientas de razonamiento automático) o la ingeniería (en visión por ordenador, en modelado de sistemas globales de navegación por satélite o en el desarrollo de esquemas de mecanizado CNC).

Programa

Jueves, 7 de febrero (tarde)

15:30 – 16:00	Ana Romero (Universidad de La Rioja) *Generalized spectral sequences for computing multipersistence*
16:00 – 16:30	Ana Marco García (Universidad de Alcalá) *Accurate algorithms for computing the bidiagonal decomposition of structured tottally positive matrices*
16:30 – 17:00	María Cruz Fernández Fernández (Universidad de Sevilla) *On some multiplicities of hypergeometric systems*
17:30 – 18:00	Luis Felipe Tabera (Universidad de Cantabria) *Geometry and algorithms for tropical separation*
18:00 – 18:30	Sebastian Falkensteiner (RISC - Research Institute for Symbolic Computation) *Formal Power Series Solutions of First Order AODEs*

VIERNES, 8 de febrero (mañana)

09:00 – 09:30	Michael Barton (Basque Center for Applied Mathematics (BCAM)) *Path planning of 5-axis flank CNC machining*
09:30 – 10:00	Laureano González-Vega (Universidad de Cantabria) *On the Interference Problem for Conics and Quadrics: Formulae and Experiments*
10:00 – 10:30	Gema María Díaz Toca (Universidad de Murcia) *Transformations from Cartesian into Geodetic Coordinates*
10:30 – 11:00	David Sevilla (Universidad de Extremadura) *Surjectivity and covering of rational surfaces*
11:30 – 12:00	Sebastià Xambó (Universidad Politècnica de Catalunya) *Un paquete de Python en la nube para el tratamiento computacional de códigos correctores de errores (A Python package in the cloud for the computational treatment of error-correcting codes*
12:00 – 12:30	Tomas Recio Muñiz (Universidad de Cantabria) *The importance of being zero for partial truth detection*

Generalized spectral sequences for computing multipersistence

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Abstract. In their original setting, both spectral sequences and persistent homology are algebraic topology tools defined from filtrations with indices in the set $\mathbb{Z}$ of integer numbers. Recently, generalizations of both concepts have been proposed which originate from a different choice of the set of indices of the filtration. Multidimensional persistence, for example, is a generalized version of persistent homology for filtrations with indices in $\mathbb{Z}^n$. On the other hand, the construction of spectral sequences has been generalized to the case of filtrations over any poset.

The main goal of this work is to show that the notions of multipersistence and generalized spectral sequences are related, generalizing in this way results which are known to be valid in the case of filtrations over $\mathbb{Z}$. We will also show some algorithms and programs implemented in the Kenzo system which make it possible to compute multipersistence by using some programs devoted to the computation of generalized spectral sequences (developed by the authors in a previous work).

Joint work with Andrea Guidolin, Basque Centre for Applied Mathematics (BCAM).
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Accurate algorithms for computing the bidiagonal decomposition of structured totally positive matrices

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Abstract. The fast and accurate computation of the bidiagonal decomposition of structured strictly totally positive matrices is a current topic of research in the area of numerical linear algebra. The interest in this topic is motivated in part by the work of P. Koev, who proved in [1] that, having the bidiagonal decomposition of a non-singular totally positive matrix computed with high relative accuracy, all numerical linear algebra problems can be solved accurately for that matrix. In this work we show how the computation of the bidiagonal decomposition of certain structured strictly totally positive matrices in exact arithmetic adapting numerical linear algebra algorithms provides fast algorithms for solving structured linear systems corresponding to interpolation problems involved, for example, in curve implicitization problems [2]. The important case of Bernstein-Vandermonde matrices will be specially considered.

Referencias

[1] P. Koev, Accurate computations with totally nonnegative matrices, SIAM J. Matrix Anal. Appl. 29 (2007) 731–751.

[2] A. Marco and J. J. Martínez, Bernstein-Bezoutian matrices and curve implicitization, Theor. Comput. Sci. 377(2007) 65–72.

Joint work with: José Javier Martínez y Raquel Viaña
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On some multiplicities of hypergeometric systems

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Abstract. A-hypergeometric D-modules, also known as GKZ systems after Gelfand, Kapranov and Zelevinsky, are systems of linear partial differential equations in several complex variables that generalize classical hypergeometric equations. A rank $d \geq 1$ integer matrix $A \in \mathbb{Z}^{d\times n}$ and a parameter vector $\beta \in \mathbb{C}^d$ determine an A-hypergeometric D-module $M_A(\beta)$. The L-characteristic cycles of a D-module are invariants that provide some information about their holomorphic and formal solutions spaces. Schulze and Walther proved that the L-characteristic variety of $M_A(\beta)$ does not depend on β but the multiplicities in the corresponding L-characteristic cycle do and, when $\beta \in \mathbb{C}^d$ is generic (more precisely, not rank-jumping), they also provided explicit formultas for them. We compute these multiplicities for any parameter $\beta \in \mathbb{C}^d$ in a combinatorial way and prove some consequences of this computation. In particular, we apply these results to the study of the irregularity of $M_A(\beta)$ along coordinate hyperplanes.

Joint work with Christine Berkesch (University of Minnesota).
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Geometry and algorithms for tropical separation

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Abstract. Contrary to the classical case, the complement a tropical hyperplane in R^n has $n+1$ connected components. So, a tropical hyperplane can separate up to $n+1$ (tropical) convex bodies. In this talk, we will provide different notions of separating hyperplanes and show that, for a reasonable definition of separation, the set of separating hyperplanes of k tropical polytopes is itself a tropically convex set. We will also show some of the computational complexity issues that appear when computing a tropical separating hyperplane.

Joint work with: Francisco Santos, Thorsten Theobald
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Formal Power Series Solutions of First Order Algebraic Ordinary Differential Equations

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Abstract. Let $\mathbb{K}$ be an algebraically closed field of characteristic zero. Given a first order autonomous algebraic ordinary differential equation, i.e. an equation of the form

$$F(y, y') = 0 \text{ with } F \in \mathbb{K}[y, y'],$$

we present a method to compute all formal power series solutions. Furthermore, by choosing for instance $\mathbb{K} = \mathbb{C}$, the computed formal power series solutions are indeed convergent in suitable neighborhoods.
By considering y and y' as independent variables, F implicitly defines an affine plane curve where local parametrizations can be computed, see e.g. [2]).
We show a sufficient and necessary condition on such a local parametrization to obtain a formal power series solution of the original differential equation by substitution. This leads to a complete characterization of initial values with respect to the number of distinct solutions extending them. Moreover, by choosing a particular initial value, we give an algorithm computing the coefficients of all solutions starting with this initial value up to an arbitrary order.

Referencias

[1] S. Falkensteiner and J.R. Sendra, *Formal Power Series Solutions of First Order Autonomous Algebraic Ordinary Differential Equations*. published in arXiv: 1803.04731, 2018.

[2] R.J. Walker, *Algebraic curves*. Princeton University Press, Princeton, 1950.

Joint work with J. Rafael Sendra
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Path planning of 5-axis flank CNC machining

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Abstract. Computer Numerically Controlled (CNC) machining is the leading manufacturing technology and this talk will focus on the finishing stage called *flank milling*. In flank milling, the milling tool, typically cylindrical or conical, is desired to have a tangential contact with the reference surface throughout its motion. Moreover, at this stage of machining, high accuracy of few micrometers for objects of size of tens of centimeters is needed and therefore the path-planning algorithm have to be carefully designed to respect synergy between the geometry of the milling tool and the input object. I will discuss two recent projects that look for the best initialization of a conical milling tool [1] and the sequential path-planning algorithm [2]. Finally, I will discuss future research directions towards machining with custom-shaped milling tools.

Referencias

[1] Bo P., Bartoň M., Pottmann H. Automatic fitting of conical envelopes to free-form surfaces for flank CNC machining, *Computer Aided Design* 91, 84–94, 2017.

[2] Calleja A., Bo P., González H., Bartoň M., López De Lacalle N. Highly-accurate 5-axis flank CNC machining with conical tools, *Journal of Advanced Manufacturing Technology* 97, 1605 – 1615, 2018.

Transformations from Cartesian into Geodetic Coordinates

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Abstract. There is a large number of articles in the literature on the transformation from Cartesian coordinates to Geodetic coordinates, most of them on a biaxial ellipsoid, with reference to Earth. This is a basic problem in geodesy and astronomy, which appears in such important applications as GPS positioning and map projections. In this talk, we present new direct symbolic-numerical algorithms for the transformation from Cartesian to Geodetic coordinates considering both the general case of a triaxial ellipsoid and the particular one of a biaxial ellipsoid. The problem is always reduced to finding a particular positive root of a particular polynomial. By using symbolic tools (Descartes' rule of signs and Sturm-Habicht coefficients mainly), we will prove the existence of these roots. We will also compare our methods with those introduced in [1, 3, 4]. Also, if time permits, we will introduce the same transformation on a biaxial hyperboloid. Our approach improves the solution presented in [2], being reduced to a few evaluations of symbolic expressions.

Referencias

[1] Feltens J. *Vector method to compute the cartesian* (X, Y, Z) *to geodetic* (ϕ, λ, h) *transformation on a triaxial ellipsoid.* Journal of Geodesy, 83(2):129–137, 2009.

[2] Feltens J. *Hyperboloidal coordinates: transformations and applications in special constructions.* Journal of Geodesy,85(4): 239–254, 2011.

[3] Ligas M. *Cartesian to geodetic coordinates conversion on a triaxial ellipsoid.* Journal of Geodesy, 86(4):249– 256, 2012.

[4] Vermeille H. *Direct transformation from geocentric to geodetic coordinates.* Journal of Geodesy, 76:451–454, 2002.

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On the Interference Problem for Conics and Quadrics: Formulae and Experiments

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Abstract. The problem of detecting when two moving conics or quadrics overlap is of interest to robotics, CAD/CAM, computer animation, etc. By analysing symbolically the sign of the real roots of the characteristic polynomial of the pencil defined by the two considered conics or quadrics, we introduce and analyse new closed formulae characterising when they are separate, touch each other externally or have a common interior point (i.e. overlap). These formulae involve a minimal set of polynomial inequalities depending only on the entries of the matrices defining the two considered conics or quadrics, need only to compute the characteristic polynomial of the pencil they define and do not require the computation of the intersection points between them.

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Surjectivity and covering of rational surfaces

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Abstract.
Parametric representations of structured surfaces, like ruled surfaces, surfaces of revolution or swept surfaces, are often used in computer graphics, CAD/CAM, and surface/geometric modeling (see e.g. [1]). Unfortunately, it is possible, and in some cases common, that parametrizations of rational objects are not surjective. In the case of curves, it is known that at most one point may be missed, see [2].
In this talk we will present a survey of results in [2, 3, 4, 5, 6]. Given a parametrization, one may try to determine first which points of the surface are not in its image. A change of parameter may result in a new parametrization that covers those points (and new ones may be missed, in turn). Another approach is to find a finite set of parametrizations associated to the given one and such that the union of the images is the whole surface.

Referencias

[1] Marsh D. Applied geometry for computer graphics and CAD. 2nd ed. Springer; 2005.

[2] Sendra JR, Winkler F, Pérez-Díaz S. Rational algebraic curves: A computer algebra approach. In series algorithms and computation in mathematics, vol. 22. Heidelberg: Springer-Verlag; 2007.

[3] Caravantes J, Sendra JR, Sevilla D, Villarino C. On the existence of birational surjective parametrizations of affine surfaces. J. Algebra 501 (2018), 206–214.

[4] Sendra JR, Sevilla D, Villarino C. Covering rational ruled surfaces. Math. Comp. 86 (2017), no. 308, 2861–2875.

[5] Sendra JR, Sevilla D, Villarino C. Missing sets in rational parametrizations of surfaces of revolution. Comput.-Aided Des. 66 (2015), 55–61.

[6] Sendra JR, Sevilla D, Villarino C. Some results on the surjectivity of surface parametrizations. Computer algebra and polynomials, 192–203, Lecture Notes in Comput. Sci., 8942, Lecture Notes in Comput. Sci., Springer, Cham, 2015.

Joint work with Jorge Caravantes, J. Rafael Sendra and Carlos Villarino.
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A Python package for the computational treatment of error-correcting codes

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Abstract. The core of the computational system to be presented is the result of coding in Python 3.$*$ the Wiris system used as the computational side of [1] and it is meant to cover the computational needs of the second edition of that book, [2]. The distinctive features of the new system will be explained and illustrated: 1) Improved and extended treatment of the materials in the first edition; 2) Coverage of the computational services for the added chapters (Code-based post-quantum cryptography, Convolutional codes, and Codes in quantum computing); 3) Flexibility to extend the system to deal with other computational tasks, as in the parallel project [3].

Referencias

[1] S. Xambó-Descamps: *Block error-correcting codes. A computational primer.* Universitext, Springer-Verlag, Berlin, 2003, VI + 265 pp.

[2] S. Xambó-Descamps, with the collaboration of N. Sayols: *Error-correcting codes – A computational primer.* Springer, 2019. See the web page CC2, whose declared purpose is "to provide free links to the friendly materials (Python files and Jupyter notebooks) produced to endow the subject matter covered in the book with powerful computational companions".

[3] S. Xambó-Descamps, with the collaboration of J. M. Miret and N. Sayols: *Intersection Theory and Enumerative Geometry – A Computational Primer.* Springer, 2019. See the web page ITEG, whose role is similar, for intersection theory and enumerative geometry, to the role of the web page in the previous reference for error-correcting codes.

Joint work with Narcís Sayols

The importance of being zero for partial truth detection

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Abstract. We present a deterministic algorithm for deciding if a polynomial ideal, with coefficients in an algebraically closed field K of characteristic zero, of which we know just some very limited data, namely: the number n of variables, and some upper bound for the geometric degree of its zero set in K^n, is or not the zero ideal. The algorithm performs just a finite number of decisions to check whether a point is or not in the zero set of the ideal. The role of this technique in the context of automated theorem proving of elementary geometry statements, is presented. In particular, we emphasize the relevance of testing if some pair of elimination ideals is zero, for the automatic handling of propositions that are simultaneously true and false over some relevant collections of instances. A rigorous, algorithmic criterion is presented for detecting such cases, and its performance is exemplified through the implementation of this test on the dynamic geometry program *GeoGebra* (see, for example the variant of Thales theorem as stated in the Automated Geometer, `http://htmlpreview.github.io/?https://github.com/kovzol/ag/blob/master/automated-geometer.html`).

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